## Impact Objectives

- Research analogue-digital mixed integrated circuit configuration using number theory
- Develop techniques for designing mixed-signal circuits by linking number theory and circuit design strategies

# Smart mathematics leads to sophisticated circuits

***Professor Haruo Kobayashi** and **Assistant Professor Anna Kuwana** tell us about their work on using number theory to investigate analogue-digital mixed integrated circuit configurations*

*Professor Haruo Kobayashi*

*Assistant Professor Anna Kuwana*

**What inspired you to become involved in this field?**

**HK:** I studied Information Physics at The University of Tokyo and Electrical Engineering at UCLA. After some industrial experiences and obtaining my PhD from Waseda University, I joined Gunma University in 1997 and have been working here since. I am now a professor within the Division of Electronics and Informatics, and there I have conducted many research projects collaborating with semiconductor and electronic measurement instrument companies.

**AK:** I completed my PhD at Ochanomizu University in 2011. I spent the next seven years as a lecturer at Ochanomizu before moving on to Gunma University as an Assistant Professor in Electronics and Informatics. My research interests focus around the area of computational fluid dynamics as well as a wide range of information and computer simulation technologies.

**Your current research is focused on studying analogue-digital mixed integrated circuit configuration using number theory. Can you explain what this involves?**

**HK:** Traditionally, the design of circuits that include analogue elements (such as analogue or mixed-signal circuits) is considered a bit "artsy", and they are often key elements in electronic systems to differentiate electronic products in industry. However, we argue that mathematics can be used to contribute towards a more scientific approach to designing innovative mixed-signal circuits. Especially, we focus on the number theory or integer theory, and it is well-known that statements of number theory theorems are often simple even though their proofs are very difficult, such as Fermat's Last Theorem. However, for their application to mixed-signal circuit design, we need only the theorem statements, not the proofs. This means we are able to take this approach even though we are not mathematicians.

**AK:** Number theory focuses on the study of integers and integer-valued functions. It falls within the realm of pure mathematics. Digital circuits certainly work well with number theory as they contain set integer values. Mixed-signal circuits include both analogue and digital circuits and so can at least partly be considered using mathematical techniques. We seek to use mathematics to contribute towards a smarter configuration of digital circuit parts.

**What are some of the big gaps in knowledge you are hoping to address through your own studies?**

**HK:** Currently, circuit design is often conducted based on existing experience and intuition of mature circuit designers and researchers. However, without robust training of new designers who learn the ropes, it would be all too easy for this skillset to be lost and further development of these circuits to be halted. If we can create a systematic design method for advancing new circuit designs, then it would be possible for anyone aware of the method to design good quality circuits without the need for previous experience or intuition. We believe that this would not only drive progress in circuit design but also give this area stability in ensuring that there is no shortage of creative designers.

**What are the main goals of this particular research project?**

**AK:** We aim to develop techniques for designing and coming up with new mixed-signal circuits by linking number theory and circuit design strategies. We also hope to extend our approach to many kinds of analogue and mixed-signal circuits.

**How do you think your studies will lead to real-world applications?**

**AK:** Circuits are a vital part of a technological society, and it is our hope that our work leads to improved circuit design across the board. Actual integrated circuit design, simulation, chip implementation and measurement can be used for many real-time applications and there is really no limit to how far and wide the impact of this may be felt. ▶

# Innovative circuit design by mathematics

*While classic mathematical theories may not seem like a logical partner for driving development in circuit design, researchers at **Gunma University** in Japan are harnessing the power of mathematics in electrical engineering*

In an electronic world, we rely heavily on circuits as the basis of our phones, televisions, computers and other electronic devices. These circuits are usually either digital, where the electrical circuit has one of two values; analogue, where the signal has a continuously variable value; or mixed-signals, which is a combination of the two.

Professor Haruo Kobayashi and Assistant Professor Anna Kuwana, from the Division of Electronics and Informatics at Gunma University in Japan, have taken the approach that harnessing the power of classical mathematics can help achieve a systematic and efficient approach to designing and coming up with new circuits that contain elements of analogue signalling. 'We are using mathematical theorems such as number theory, which focuses on integers; and the Fibonacci sequence, which is a sequence of numbers that are formed by adding the two previous numbers in the sequence and control theory, which centres on the concept of the influence of feedback on the behaviour of a system,' explains Kobayashi. This mathematical approach to circuit design is unusual but as the researchers have found, it has proven effective and yields promising results.

Analogue circuit design is typically more challenging than its digital counterpart as it requires a higher level of skill and experience to conceptualise. This means that these circuits are usually designed by hand, the process is less automated and more demanding than digital circuit design. Analogue circuit planning is therefore often regarded as an art rather than a strictly scientific process as there are currently no systematic or theoretical approaches in place to streamline the draughting process. Kobayashi and Kuwana firmly believe that 'beautiful' mathematics can provide a path towards truly great circuit design. 'Currently, there is no systematic way for deriving new mixed-signal circuit configurations; only genius designers can do this,' explains Kobayashi.

## DEVELOPING THE TOOLS FOR PROGRESS

Kobayashi and his team have used several mathematical techniques to guide transistor-level circuit and architecture-level design and improve the performance of the resultant analogue and mixed-signal circuits. 'For example, we applied the Fibonacci sequence in their exploration of redundant successive approximation register (SAR) ADC design methods for reliable, high-speed AD conversion,' outlines Kobayashi. 'We also apply number theory, statistics, coding theory, modulation, control theory and signal processing algorithms to circuit design to improve performance as well as to develop more efficient processes for designing circuit architecture,' he says. This pairing of pure mathematics theorems with electrical engineering is a key feature of their work.

> *We have shown that in equivalent time sampling, when the ratio between the input frequency and the sampling frequency is one of the metallic ratios, the waveform acquisition is very effective*

Validation of findings is a critical part of research work and Kobayashi's team project is no different. Much of the work they carry out involves a significant amount of preliminary work in considering how different mathematical solutions might benefit circuit architecture, addressing important issues such as speed and reliability. The group naturally uses theoretical analysis and a variety of simulation methods to test their findings. Simulators such as the circuit simulator (SPICE) and system simulator (MATLAB) are important aids in the modelling work.

As with much essential research these days, collaborations are also key in progressing Kobayashi and Kuwana's work. 'We have collaborated with semiconductor companies and electronic measurement instrument companies in Japan for smart circuit design and effective testing of the circuits,' describes Kuwana. 'They can design and test analogue/mixed-signal circuits effectively and systematically for their electronic products,' she says. This academic-industry partnership is mutually beneficial, bringing the Gunma University team's high level theoretical and practical knowledge together with resources and on-the-ground testing and design facilities of their industry partners.

## FACING THE CHALLENGES

The partnership of mathematics and electronics design may not seem obvious to many, and it has indeed raised some obstacles to the team's work. Knowledge of circuit design, particularly in the design of circuits that contain analogue elements, is a learned art and understanding this as well as classic mathematical concepts such as integer theory are two very different areas of expertise and theoretical knowledge. Kobayashi and Kuwana, both electrical engineers, were already well versed in circuit architecture. However, they needed to also study key concepts in pure mathematics in depth to not only identify suitable solutions to circuit design problems but also to enable them to forge the links between these two fields.

They have had a number of encouraging results, including confirmation that using SAR ADC configurations with Fibonacci sequence weights can improve the speeds and reliability of the SAR ADC. The team has also used the metallic ratio sampling method in their work. 'We have shown that in equivalent time sampling, when the ratio between the input frequency and the sampling frequency is one of the metallic ratios, the waveform acquisition is very effective. We have also derived some theoretically beautiful properties,' comments Kobayashi. The elegance of pure mathematics is something that appeals to the researchers, and they are keen to demonstrate that the beauty of mathematics can indeed lead to better electronics design and enhanced practical utility. 'Our results are theoretically beautiful and also practically useful,' observes Kuwana.

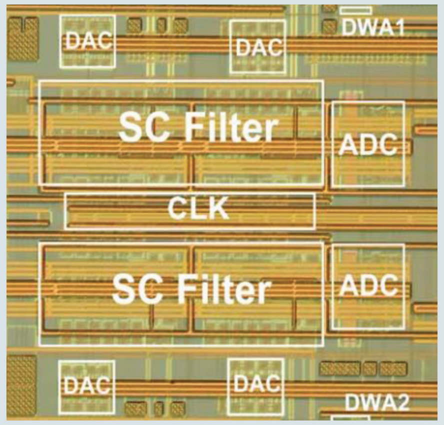
The team's research results are as follows: Fibonacci number weighted SAR ADC, Silver ratio weighted SAR ADC, Gray-code input DAC, Unary DAC layout algorithms using Magic Square, Latin Square and Euler Knight Tour, DACs based on polygonal number theorem and Goldbach conjecture, Waveform sampling using residue number theorem and metallic ratio theory, ADC histogram test using metallic ratio sampling, Time-to-digital converters based on Gray-code and residue number theory, Time-to-digital converter based on statistics, ADC based on complex number theory as well as Time-to-digital converter self-calibration using metallic ratio theory. These research results have been published in international journals. In addition, Kuwana is currently developing efficient pseudo-random signal generation algorithms for Monte-Carlo circuit simulations, using the golden-ratio sampling technique.

## BENEFITS AND THE FUTURE

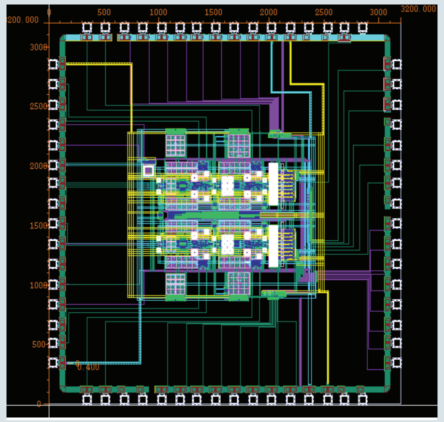
In creating the linkages between mathematical theory and circuit design, Kobayashi's group hopes to provide a systematic design methodology for designing analogue and mixed-signal circuits. This will help derive new circuits and improve the process of designing these circuits, which will help promote their development. Not only would circuit designers benefit directly from this work, but a wider global audience of electronics users will also indirectly gain as progress in circuit design can become both easier and faster.

Kobayashi's group have already been working on using various mathematical theories to guide their work and several papers have been published to share these discoveries. They have utilised control theory in operational amplifier design, harnessed the Fibonacci sequence in SAR ADC design, and considered DAC layout design employing magic square properties. 'As we reflect on the next step on our research pathway, we seek to apply number theory to testing analogue-to-digital converters as well as using number theory to time-to-digital converter linearity calibration,' highlights Kobayashi.

The team's unique approach to analogue and mixed-signal circuit design has so far offered promising results. With their industry collaborations, they are able to leverage knowledge and combine it with the ability to build and test circuits to validate their theories. This is an important aspect to this work and they have made encouraging progress thus far. The potential benefits to circuit designers as well as the electronics industry and to the end users are tremendous if this potential for faster, smoother design workflows is realised.



*Chip photo of analogue-to-digital converter*


*Chip layout of analogue-to-digital converter*

## Project Insights

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**BIO**
**Professor Haruo Kobayashi** joined Gunma University in 1997 and is now based at the Division of Electronics and Informatics. His research interests include mixed-signal integrated circuit design and testing, and signal processing algorithms. He received the Yokoyama Award in Science and Technology in 2003.

**Assistant Professor Anna Kuwana** joined Gunma University in 2018 in the Division of Electronics and Informatics. Her research interests include Computational Fluid Dynamics as well as a wide range of information and computer simulation technologies.

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